Gravitational N-body Simulations

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ABSTRACT

Gravitational *N*-body simulations, that is numerical solutions of the equations of motions for N particles interacting gravitationally, are widely used tools in astrophysics, with applications from few body or solar system like systems all the way up to galactic and cosmological scales. In this article we present a summary review of the field highlighting the main methods for N-body simulations and the astrophysical context in which they are usually applied.

1 Introduction

The underlying dynamics relevant in the astrophysical context for of a system of N particles interacting gravitationally is typically Newton's law plus, in case, an external potential field (see however below for a discussion of N-body simulations in general relativity). The force \vec{F}_i acting on particle *i* of mass m_i is:

$$\vec{F}_{i} = -\sum_{j \neq i} G \frac{m_{i} m_{j} (\vec{r}_{i} - \vec{r}_{j})}{|\vec{r}_{i} - \vec{r}_{j}|^{3}} - \vec{\nabla} \cdot \phi_{ext}(\vec{r}_{i}),$$
(1)

where $G = 6.67300 \cdot 10^{-11} m^3 kg^{-1} s^{-2}$ is the gravitational constant, and ϕ_{ext} is the external potential. The problem is thus a set of non-linear second order ordinary differential equations relating the acceleration $\partial^2 \vec{r_i}/\partial t^2 = \vec{F_i}/m_i$ with the position of all the particles in the system.

Once a set of initial condition is specified (for example the initial positions \vec{r}_i and velocities $\vec{v}_i \equiv \partial \vec{r}_i / \partial t$ of all particles) it exists a unique solution, analytical only for up to two bodies, while larger N require numerical integration (e.g. see Press et al. 2007). However special care must be employed to ensure both accuracy and efficiency. In fact, the gravitational force (eq. 1) presents a

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singularity when the distance of two particles approaches 0, which can lead to arbitrarily large relative velocities. In addition, given the non-linear nature of the equations, the singularities are movable, that is they depend on the specific choice of initial conditions. In contrast, all singularities in linear ordinary differential equations are independent of initial conditions and thus easier to treat. Therefore constant timestep methods are unable to guarantee a given accuracy in the case of gravitational dynamics and lead to unphysical accelerations during close encounters, which in turn may create unbound stars. A shared adaptive timestep scheme can correctly follow a close encounter, but the price is paid in terms of efficiency as all the other particles of the system are evolved on the timescale of the encounter, which may be several orders of magnitude smaller than the global timescale, resulting essentially in a freezing of the system.

The singularity may be avoided by introducing a smoothing length in Eq. 1 (e.g. see Aarseth 1963), that is by modifying the gravitational interaction at small scales. For example:

$$\vec{F}_i = -\sum_{j \neq i} \frac{Gm_i m_j (\vec{r}_i - \vec{r}_j)}{(|\vec{r}_i - \vec{r}_j|^2 + \epsilon^2)^{3/2}},\tag{2}$$

where $\epsilon > 0$ is the softening, or smoothing length, that is a typical distance below which the gravitational interaction is suppressed. To minimize the force errors and the global impact of the softening for distances larger than ϵ , finite size kernels that ensure continuous derivatives of the force may be employed (e.g., see Dehnen 2001). This strategy effectively suppresses binary formation and strong gravitational interactions, but at the price of altering the dynamics of the system.

The computational complexity of the numerical solution of a N-body system for a fixed number of timesteps scales as N^2 , as the evaluation of the force on each particle requires to take into account contributions from all other members of the system. For example, considering a single state of the art cpu core (speed ≈ 5 GFlops), a single force evaluation through a direct method would require about 1 second for a system with $N = 10^4$ particles (assuming 10 floating point operations per pair of particles) and more than a week for $N = 10^7$.

The arbitrarily large dynamic range in the unsoftened dynamics and the expensive evaluation of the force have led to the development of a wide number of numerical techniques aimed at obtaining a reliable numerical solution with the minimum amount of computational resources, depending on the astrophysical problem of interest. Here we start by discussing the different astrophysical contexts where N-body simulations are routinely employed and we then present the state of the art techniques for these problems.

2 Astrophysical domains and timescales

N-body simulations are applied to a wide range of different astrophysical problems so that the most appropriate technique to use depends on the specific context, and in particular on the timescale and collisionality of the problem.

2.1 Timescales, Equilibrium and Collisionality

A system of N particles interacting gravitationally with total mass M and a reference dimension R (for example the radius containing half of the total mass) reaches a dynamic equilibrium state on a timescale comparable to a few times the typical time (T_{cr}) needed for a particle to cross the system $(T_{cr} \approx 1/\sqrt{GM/R^3})$. This is the response time needed to settle down to virial equilibrium, that is 2K/|W| = 1, where K is the kinetic energy of the system: $K = 1/2 \sum_{i=1,N} m_i |\vec{v_i}|^2$, and W is its potential energy: $W = -1/2 \sum_{i\neq j} Gm_i m_j / |\vec{r_i} - \vec{r_j}|$ (assuming no external field). If the system is initially out of equilibrium, this is reached through mixing in phase space due to fluctuations of the gravitational potential, a process called violent relaxation (Lynden-Bell 1967).

Once the system is in dynamic equilibrium a long term evolution is possible, driven by two-body relaxation. Energy is slowly exchanged between particles and the system tends to evolve toward thermodynamic equilibrium and energy equipartition. The timescale (T_{rel}) for this process depends on the number of particles and on the geometry of the system: $T_{rel} \propto N/\log(0.11N)T_{cr}$ (e.g. see Spitzer 1987). N-body systems such as galaxies and dark matter halos have a relaxation time much longer than the life of the Universe and are thus considered collisionless systems. Smaller systems, such as globular and open clusters, are instead collisional, as the relaxation time is shorter than their age. Two body relaxation is also suppressed when one particle in the system dominates the gravitational potential, such as in the case of solar system dynamics, where planets are essentially quasi-test particles.

Close encounters between three or more particles not only contribute to energy exchange, but can also lead to the formation of bound subsystems (mainly binaries). The formation and evolution of a binary population is best followed through direct, unsoftened, N-body techniques.

A self-gravitating N-body system made of single particles has a negative specific heat, that is it increases its kinetic energy as a result of energy losses (Lynden-Bell & Wood 1968). This is a consequence of the virial theorem and qualitatively it is analogous to the acceleration of a Earth artificial satellite in presence of atmospheric drag. A negative specific heat system is thermodynamically unstable and over the two body relaxation timescale it evolves toward a gravothermal collapse, creating a core-halo structure, where the core progressively increases its concentration, fueling an overall halo expansion. The collapse is eventually halted once three body interactions lead to the formation of binaries. The so called "core collapsed globular clusters" are considered to be formed as a result of this mechanism.

2.2 Mean field approach: the Boltzmann equation

A system of N particles interacting gravitationally defines a 6N+1 dimensional phase space given by the N position and velocity vectors associated to each particle at each time t. The solution of the N-body problem defines a trajectory in this phase space. If the number of particles is large enough, that is if the two body relaxation time is long compared to the time-frame one is interested in, then a statistical description of the problem is possible. This allows us to pass from a 6N+1 to a 6+1 dimension phase space. The idea is to construct a mean field description of the dynamical system in terms of a single particle distribution function $f(\vec{x}, \vec{v}, t)$, where $f(\vec{x}, \vec{v}, t)d^3xd^3v$ is proportional to the probability of finding a particle in a 6D element of volume d^3xd^3v centered around position \vec{r} and velocity \vec{v} at time t. Within this simplified framework the knowledge of the distribution function uniquely defines all the properties of the system. The dynamic is described by the collisionless Boltzmann equation, which derives essentially from the Liouville theorem:

$$\frac{Df}{Dt} = \frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{x}} - \frac{\partial \phi_T}{\partial \vec{x}} \cdot \frac{\partial f}{\partial \vec{v}} = 0, \qquad (3)$$

where the total potential field $\phi_T = \phi_{ext}(\vec{x}, t) + \phi(\vec{x}, t)$ is the sum of an external potential plus the self-consistent field $\phi(\vec{x}, t)$ defined from the distribution function itself through the solution of the Poisson equation:

$$\nabla^2 \phi(\vec{x}, t) = 4\pi G \rho(\vec{r}, t), \tag{4}$$

where $\rho(\vec{r},t) = \int f(\vec{x},\vec{v},t) d^3v$.

Given its high dimensionality (6+1), the collisionless Boltzmann equation is usually solved by sampling the initial distribution function and then by evolving the resulting N-body system by means of a numerical method that suppresses two body interactions at small scales. The interaction is softened not only for computational convenience to limit the maximum relative velocity during close encounters but especially to prevent artificial formation of binaries. This is because a simulation particle in a collisionless run represents in reality an ensemble of real particles (e.g. galaxies contain 10^{11} stars but simulations typically use only $N \in [10^6 : 10^9]$). Note however that two body relaxation is driven by close as well as by distant encounters, so softening does not suppress it. In principle any numerical method that has a small scale softening is appropriate for following collisionless dynamics.

A mean field description for an N-body system is possible also for collisional systems, that is when the relaxation time is comparable to or shorter than the timeframe of interest. In this case the collisionless Boltzmann equation is modified by the introduction of a collision operator C[f] on its right side:

$$\frac{Df}{Dt} = \frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{x}} - \frac{\partial \phi_T}{\partial \vec{x}} \cdot \frac{\partial f}{\partial \vec{v}} = C[f].$$
(5)

In this framework the operator C[f] describes the probability for particles to enter/leave a phase space element as a result of gravitational encounters. The collision operator C is generally constructed assuming that encounters are:

1. Markov processes, that is C depends only on the present state of the system;

- 2. local, that is only the velocity of the particles are changed and not their positions;
- 3. weak, that is the typical velocity change is much smaller than the velocity itself.

Under these assumptions Monte Carlo methods are available to solve the dynamics of the system (see next section). Applications of the collision operator include dynamics of globular clusters and of self-interacting dark matter.

2.3 Mean Field Approach: analogies and differences with fluid dynamics

The velocity moments of the Boltzmann Equation define a set of equations known as the Jeans Equations (e.g. Binney & Tremaine 2007). The first three equations of the set are formally identical to the Navier-Stokes equations for a self-gravitating gas and, like in the fluid-dynamics analogy, express the conservation of mass, momentum and energy. Therefore the numerical algorithms developed to follow the dynamics of N-body systems find a wide application also in the context of fluid-dynamics, with one important example being the Smoothed Particle Hydrodynamics (SPH) method (Gingold & Monaghan 1977). The fundamental difference between the two cases is that the Jeans equations are derived in the limit of a collisionless system, while the Navier-Stokes equations assume a highly collisional system, with the mean free path of a particle approaching zero. For fluids, this leads to the definition of an equation of state, which closes the Navier-Stokes equations. The Jeans Equations are instead an infinite open set, where the *n*-th velocity moment depends on the *n*-th+1 moment.

2.4 Astrophysical domains

Based on the previous considerations about collisionality and timescales, four main astrophysical domains for N-body simulations can be identified, each requiring a different numerical technique to guarantee maximum performance and accuracy:

Celestial mechanics (solar and extrasolar planetary systems). Here a single body dominates the gravitational field and all the other objects move almost like test particles, subject to reciprocal perturbations. In this framework very high accuracy is required to correctly evaluate the perturbative terms and to avoid being dominated by numerical noise such as time discretization and round-offs errors.

Dense stellar systems, such as open clusters and globular clusters. These collisional systems made of components of roughly equal mass present a rich dynamics, with multiple close encounters of stars. The evolution requires to be followed on a relaxation timescale with a correct description of the short range interactions.

Sphere of influence of a massive BH at the center of a stellar system. The sphere of influence of a BH is the volume within which the gravity of the BH dominates over that of the other particles. The situation resembles that of solar system dynamics, but here given the very high density of stars two body encounters are frequent, making the problem a difficult hybrid between the two previous cases. In addition, Post Newtonian physics may need to be included if high accuracy is required in the proximity of the BH.

Galaxy dynamics and cosmology. Galaxies, and especially dark matter halos, are constituted by a very large number of particles, so that their dynamics can be well described in terms of a mean field. Close encounters are not important and softening is usually employed in these N-body simulations to avoid the unphysical formation of binaries. Within this class, Self-Interacting Dark Matter Particles need a special mention: if dark matter halos are made of Weakly Interacting Massive Particles, then their dynamics can be modified by nongravitational self-interactions, especially effective at the center of cuspy dark halos. The dynamics of such a system is described by the Collisional Boltzmann Equation, which can be approximately solved using Fokker-Plank methods.

3 Newtonian gravity: methods

The history of N-body simulations starts with a pioneering attempt by Holmberg (1941), who followed the evolution of a 37 particle system, where the force was calculated using lightbulbs and galvanometers (taking advantage of the same r^{-2} scaling of electromagnetic and gravitational interactions). Computer simulations started in the early sixties using up to 100 particles (e.g. see von Hoerner 1960 and Aarseth 1963) and had their full bloom in the eighties with the development of fast and efficient algorithms to deal with collisionless systems, such as particle-mesh codes (see Hockney & Eastwood 1988 and references therein) and the tree method (Barnes & Hut 1986). At the same time regularization techniques were developed to deal with close encounters and binary dynamics in the case of direct simulations of a collisional system (e.g. see Aarseth's NBODY-X code series based on KS and chain regularization - Aarseth 2003 and references therein). These algorithm advancements were coupled with tremendous progresses in the hardware, with the cpu speed growing exponentially. In addition to parallelization of serial codes, the field advanced also thanks to special purpose hardware, such as the GRAPE (Makino et al. 1997). Today's (2008) N-body simulations are performed with up to $N = 10^5$ (e.g. see Baumgardt & Makino 2003) for direct integration codes over a two-body relaxation timescale and up to $N = 10^{10}$ for collisionless dynamics/cosmology (e.g. see the Millennium Run - Springel et al. 2005). In the context of planetary dynamics, self-gravitating systems of disk/ring particles with $N \approx 10^6$ can be followed over hundreds of dynamical times (e.g. Richardson et al. 2000). Major breakthroughs are also expected in the near future thanks both to the next generation GRAPE-DR and to double precision graphic processing units, which provide extremely cost competitive high performance computing capabilities.

3.1 Direct methods

Direct methods do not introduce approximations in the solution of the equations of motions and thus deliver the highest accuracy at the price of the longest computation time, of order $O(N^2)$ per timestep. Integration is performed using adaptive (individual) timesteps and commonly a fourth order Hermite integrator. Close encounters and bound subsystems are treated exactly in terms of Kustaanheimo-Steifel transformations. These essentially consist in transformations of coordinates using a perturbative approach over the analytical two body solution. If more than two particles have a strong mutual interaction, then a chain regularization strategy (Mikkola 1990) can be used, which consists in recasting the problem in terms of a series of separate Kustaanheimo-Steifel interactions. A state of the art, publicly available, serial direct N-body integrator is Aarseth's NBODY6. Even with this specialized software, the number of particles that can be effectively followed for timescales comparable to the Hubble time is limited. For example, if one is interested in the dynamical evolution of globular clusters, currently about N = 20000 is the practical limit for a serial run, as such a run takes about 1000 cpu hours. A run with 10^6 particles carried out for a similar number of relaxation times T_{rel} would require about 10^8 cpu hours. The algorithm can be parallelized, but in practice load imbalances may saturate the gain in efficiency, so some of the most cpu demanding simulations have been carried out on special purpose hardware, such as the GRAPE, where the chip architecture has been optimized to compute gravitational interactions, thus delivering Teraflops performance.

3.2 Tree codes

The tree code method (Barnes & Hut 1986) provides a fast, general integrator for collisionless systems, when close encounters are not important and where the force contributions from very distant particles does not need to be computed at very high accuracy. In fact, with a tree code, small scale, strong interactions are typically softened (but see McMillan & Aarseth 1993), while the potentials due to distant groups of particles are approximated by multipole expansions about the group centers of mass. The resulting computation time that scale as O(Nloq(N)) but the approximations introduce some (small) errors. The errors in the long-range component of the gravitational acceleration are controlled by a single parameter (the so called opening angle) that determines how small and distant a group of particles must be to use the approximation. This strategy works well to keep the average force error low, but a worst case scenario analysis highlights that unbound errors can arise for rare, but astrophysically reasonable configurations, such as that of the classic "exploding galaxy" (Salmon & Warren 1994). In addition, force errors from the tree code may lead to violation of momentum conservation. Typical implementations of the tree code expand the potentials to quadrupole order and construct a tree hierarchy of particles using a recursive binary splitting algorithm. The tree does not need to be recomputed from scratch at every timestep, saving significant cpu time. Systems with several hundred thousands of collisionless particles can be easily simulated on a GFlops workstation for a Hubble time using this method.

3.3 Fast Multipole Methods

A standard tree code implementation does not take advantage of the fact that nearby particles will be subject to a similar acceleration due to distant groups of particles. The Fast Multipole Method (Greengard & Rokhlin 1987) exploit this idea and uses a multipole expansion to compute the force from a distant source cell within a sink cell. This additional approximation of the gravitational interaction was claimed to reduce the complexity from O(Nlog(N)) to O(N), but the exact scaling seems implementation dependent and has been debated in the literature (e.g. see Dehnen 2000 and references therein). One advantage of the fast multipole method is that the symmetry in the treatment of sink and source cells with respect to the multipole expansion can guarantee an exact conservation of the momentum. Recent successful implementations of fast multipole codes or hybrids with tree code scheme, include Dehnen's Cartesian expansion scheme (the GyrfalcON code- Dehnen 2000) and PKDGRAV (Stadel 2001).

3.4 Particle-mesh codes

The particle mesh method represents another route to speed up direct force evaluation for collisionless systems. In this case the gravitational potential of the system is constructed over a grid starting from the density field and by solving the associated Poisson equation. Particles do not interact directly between each other but only through a mean field. The method essentially softens the gravitational interactions at small scales, that is below the cell length. The density field is constructed using a kernel to split the mass of the particles to the grid cells around the particle position. The simplest choice is to assign all the mass to a single cell, but this leads to significant force fluctuations, which can be reduced using a cloud in cell (8 points) or a triangular shaped cloud (27 points) kernel. The Poisson equation is typically solved using a Fast Fourier Transform, but other grid methods such as successive overrelaxation can also be used - e.g. see Bodenheimer et al. (2007). The deriving force, defined on the grid, is then assigned back to the particles using the same kernel employed for the density field construction, in order to avoid spurious self forces. The method achieves a linear complexity in the number of particles and $(O(N_a log(N_a)))$ in the number of grid cells (this latter scaling is that of the FFT method). The price to pay is in terms of short range accuracy as the force is a poor approximation of Newton's law up to several grid spacing of distance.

3.5 Adaptive Mesh Refinement method

The dynamic range of particle-mesh codes can be increased by using an adaptive rather than a static grid to solve the Poisson Equation. In the Adaptive Mesh Refinement (AMR) method the grid elements are concentrated where a higher resolution is needed, for example around the highest density regions. One possibility to obtain an adaptive resolution is to first construct a low-resolution solution of the Poisson Equation and then to progressively refine regions where the local truncation error (estimated through the Richardson extrapolation) is highest. A multigrid structure needs to take into account issues such as matching the solution at the grid interfaces. AMR codes are well suited for cosmological simulations (e.g. see the ENZO code, Bryan & Norman 1998).

3.6 Self consistent field methods

A variant over the Particle Mesh code is the expansion of the density and potential of the system in terms of a basis of orthogonal eigenfunctions. Clutton-Brock (1972) was one of the first to apply this idea in stellar dynamics, while a modern implementation is that of Hernquist & Ostriker (1992). This method guarantees at fixed computational resources a higher accuracy than the tree code and the particle mesh algorithms, provided that the set of basis function is appropriately selected. This limits in practice a general application of the method, which remains however very competitive for the study of the dynamical stability of collisionless systems constructed from distributions functions models.

3.7 P3M and PM-Tree codes

In order to increase the force resolution of particle mesh codes it has been proposed to couple a mean field description on large scales with a direct, softened, treatment of the gravitational interactions on distances of the order of or below a few grid spacing. This method is called P^3M (Hockney & Eastwood 1988): Particle-Particle-Particle-Mesh and efficiently increases the dynamic range of the parent PM algorithm. However in presence of strong clustering a large number of particles will interact directly between each other, slowing down significantly the computation to $O(N^2)$. This problem can be resolved by using adaptive meshes, so that the spatial resolution is refined in regions of high density. Adaptive P^3M codes have a computational cost which scales as O(Nlog(N)), like in a tree code. Finally another possibility is to resort to a tree code for the short range force evaluation leading to a hybrid PM-Tree scheme. These methods are generally extremely well suited for cosmological simulations, for example see Gadget2 (Springel 2005).

3.8 Celestial mechanics codes

Computational Celestial Mechanics refers to a series of methods targeted at studying the dynamics of small N systems ($N \leq 20$). The smallest non trivial N is N=3, that is the three body problem, which has many applications ranging from space flight to planets satellite motions and to binary-single stars encounters. Celestial mechanics requires extremely high precision given the chaotic nature of the N-body problem. Numerical methods are based on the use of

local system of coordinates, to fight round-off errors in systems with a wide dynamic range, such as in the study of star-planet-satellite problems, as well as on the variational equations formalism and on perturbation theory to take advantage of the analytical, unperturbed motion of planets in the gravitation field of their star (e.g. see Beutler 2005). In this context symplectic integrators are widely used (e.g. see Wisdom & Holman 1991; Leimkuhler & Reich 2005).

4 Mean Field Methods

As an alternative to particle based N-body methods, the dynamics of a system of particles interacting gravitationally can be followed by solving the time dependent Boltzmann Equation coupled with the self-consistent Poisson equation.

4.1 Grid based solvers for the Collisionless Boltzmann Equation

This approach can take advantage of standard computational methods developed to solve partial differential equations, such as successive over-relaxation and conjugate gradient methods. However it requires to solve a highly dimensional (6D+time) non-linear system of partial differential equations. In general, the bottleneck is thus the very large amount of memory needed (for example, Terabites just to have a moderate resolution grid with 100 elements in each dimension). However this method is competitive if the astrophysical problem of interest presents symmetries that reduce the number of dimensions needed in the model. For example, in the case of globular cluster dynamics a very good approximation can be obtained via a 3 dimensional model by assuming spherical symmetry in the position space (1D) and radial anisotropy in the velocity space (2D).

4.2 Fokker-Planck and Monte Carlo methods

These methods solve the collisional Boltzmann equation starting from a given distribution function and by following test particles in the six dimensional position-velocity phase space. At each timestep the velocity of the particles is perturbed by random fluctuations accordingly to the assumed form for the collision operator C[f], which depends on computed cross sections for two, three and four body encounters. The complexity of Monte Carlo codes is linear with the number of particles and thus a realistic number of particles can be used for simulations of collisional systems with $N > 10^5$ with a serial code. The method is ideal for exploring grids of initial conditions, after proper validation through comparison with direct integration (e.g. see Heggie et al. 2006).

4.3 Beyond Newton: strong gravitational fields

In presence of a strong gravitational field, such as that in the proximity of the event horizon of a black hole, N-body simulations cannot be based on Newtonian physics, but must take into account a general relativity framework. As a numerical solution of the Einstein equation is extremely challenging, Post-Newtonian approximations are used when the gravitational field does not deviate too much from the Newtonian case. Post-Newtonian corrections are typically good enough to treat most astrophysical problems of the dynamics of stars around a black hole. A full general relativity framework is only required to study the merging and gravitational waves emission of two black-holes (e.g. see Baker et al. 2006).

5 Hardware

An alternative approach to increase the efficiency of numerical solution of the N-body problem is to optimize the hardware. For direct simulations this approach can be very effective, thanks to the fact that the bottle neck of computation is just the evaluation of the gravitational force, which has a very simple expression. Along this route the GRAPE (GRavityPipE) concept has been extremely effective. The basic idea is to optimize a hardware pipeline to compute $(\vec{r_i} - \vec{r_j})/|\vec{r_i} - \vec{r_j}|^3$. This special purpose hardware can then be interfaced with a general purpose computer, which takes care of all the other numerical operations required to solve the equations of motions. With the GRAPE-6, the largest simulation on a collisional timescale published to date has N=131028 (Baumgardt & Makino 2003).

Another recent promising hardware development is the possibility to use Graphic Cards (GPUs) to carry out the cpu intensive force evaluation. The performance of current generation of GPUs appears to be superior (in terms of Flops/\$ ratio) to that of the GRAPE6 series (Portegies-Zwart et al. 2007) even if one important limitation of GPUs is that they currently operate in single precision.

6 Simulation environments

In addition to the availability of stand-alone codes, several software environments have been created that contain various tools to set up initial conditions, run simulations, and analyze and visualize their results. Some examples are NEMO, Starlab, ACS and MUSE (see below for links to their web-pages).

7 Suggested readings

7.1 Books

• "Computer Simulation Using Particles" Hockney, R.W. and Eastwood, J.W. 1988

- "Gravitational N-Body Simulations: Tools and Algorithms" Aarseth, S. 2003
- "The Gravitational MillionBody Problem" Heggie, D.C. and Hut, P. 2003
- "Methods of Celestial Mechanics" Beutler, G. 2005
- "Numerical Recipes" Press, W.H., Teukolsky, S.A., Vetterling, W.T. and Flannery B.P. 2007
- "Numerical Methods in Astrophysics: An Introduction" Bodenheimer, P., Laughlin, G.P., Rozyczka, M. and Yorke, H.W. 2007

7.2 Review articles

* "Simulations of Structure Formation in the Universe" Bertschinger, E. 1998, ARA&A, 36, 599

7.3 Web Material

• "The N-body Constitution" by Lake, G, Katz, N., Quinn T. and Stadel. J. (http://www-hpcc.astro.washington.edu/old_content/siamhtml/siamhtml.html)

8 Open source codes

- Aarseth's direct integration codes: http://www.ast.cam.ac.uk/sverre/web/pages/nbody.htm
- ACS, a collection of tools and introductory texts: http://www.artcompsci.org/
- ENZO, a cosmological AMR code: http://lca.ucsd.edu/portal/software/enzo
- Gadget2, a cosmological PM-tree+SPH code (massively parallel): http://www.mpa-garching.mpg.de/gad
- Mercury (a mixed variable symplectic integrator code for planetary dynamics): http://www.arm.ac.uk/jec/home.html
- MUSE, a software framework for simulations of dense stellar systems: http://muse.li/
- NEMO collection (includes particle-grid and tree codes): http://bima.astro.umd.edu/nemo/
- Starlab (including the direct integration Kira code): http://www.ids.ias.edu/starlab/starlab.html

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